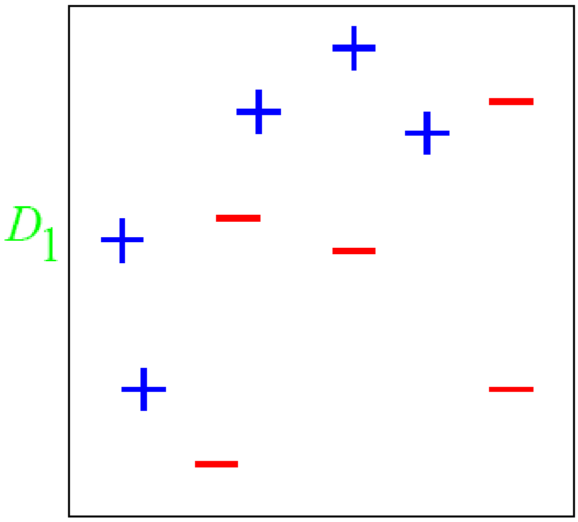
Assignment 1 (Due: Oct. 22, 2023)
(Send your solutions to our TA, Tianjun Zhang, 1911036@tongji.edu.cn)

1. (**Programming**) AdaBoost is a powerful classification tool, with which a strong classifier can be learned by composing a set of weak classifiers. In our lecture, we use a vivid example to demonstrate the basic idea of AdaBoost. Now, your task is to implement this demo.

   Training:
   There are 10 samples on a 2-D plane and information of the ith sample is given as (x_i, y_i, l_i), where (x_i, y_i) is its coordinate and l_i is its label. 10 samples are (80, 144, +1), (93, 232, +1), (136, 275, -1), (147, 131, -1), (159, 69, +1), (214, 31, +1), (214, 152, -1), (257, 83, +1), (307, 62, -1), (307, 231, -1). Weak classifiers are vertical or horizontal lines as described in our lecture. The final trained strong classifier actually is a function having the form,

   $$\text{label} = \text{strongClassifier}(x, y)$$

   Finally, test your strong classifier to verify whether it can correctly classify all the training samples.

2. (**Math**) There are n p-dimensional data points and we can stack them into a data matrix, $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^n, \mathbf{x}_i \in \mathbb{R}^{p\times 1}, \mathbf{X} \in \mathbb{R}^{p\times n}$. The covariance matrix of $\mathbf{X}$ is $\mathbf{C} = \frac{1}{n-1}\sum_{i=1}^{n}(\mathbf{x}_i - \boldsymbol{\mu})(\mathbf{x}_i - \boldsymbol{\mu})^T$, where $\boldsymbol{\mu} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_i$ (actually, it is the mean of the data points).
   1) Please prove that $\mathbf{C}$ is positive semi-definite.
   2) Based on discussions in our lecture, we know that if $\boldsymbol{\alpha}_1$ is the eigen-vector

associated with the largest eigen-value of **C**, the data projections along $\boldsymbol{\alpha}_1$ will have the largest variance. Now let's consider such an orientation $\boldsymbol{\alpha}_2$. It is orthogonal to $\boldsymbol{\alpha}_1$; and among all the orientations orthogonal to $\boldsymbol{\alpha}_1$, the variance of data projections to $\boldsymbol{\alpha}_2$ is the largest one. Please prove that: $\boldsymbol{\alpha}_2$ actually is the eigen-vector associated to **C**'s second largest eigen-value. (we can assume that $\boldsymbol{\alpha}_2$ is a unit-vector)