Assignment 3 (Due: Dec. 31, 2023)

1. (Math) Nonlinear least-squares. Suppose that $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_m(\mathbf{x})) : \mathbb{R}^n \to \mathbb{R}^m$, $\mathbf{x} \in \mathbb{R}^n, \mathbf{f} \in \mathbb{R}^m$ and some $f_i(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ is a (are) non-linear function(s). Then, the problem,

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{f}(\mathbf{x})\|_2^2 = \arg\min_{\mathbf{x}} \frac{1}{2} (\mathbf{f}(\mathbf{x}))^T \mathbf{f}(\mathbf{x})$$

is a nonlinear least-squares problem. In our lecture, we mentioned that Levenberg-Marquardt algorithm is a typical method to solve this problem. In L-M algorithm, for each updating step, at the current \mathbf{x} , a local approximation model is constructed as,

$$L(\mathbf{h}) = \frac{1}{2} \left(\mathbf{f} \left(\mathbf{x} + \mathbf{h} \right) \right)^{T} \mathbf{f} \left(\mathbf{x} + \mathbf{h} \right) + \frac{1}{2} \mu \mathbf{h}^{T} \mathbf{h}$$

$$= \frac{1}{2} \left(\mathbf{f} \left(\mathbf{x} \right) \right)^{T} \mathbf{f} \left(\mathbf{x} \right) + \mathbf{h}^{T} \left(\mathbf{J} \left(\mathbf{x} \right) \right)^{T} \mathbf{f} \left(\mathbf{x} \right) + \frac{1}{2} \mathbf{h}^{T} \left(\mathbf{J} \left(\mathbf{x} \right) \right)^{T} \mathbf{J} \left(\mathbf{x} \right) \mathbf{h} + \frac{1}{2} \mu \mathbf{h}^{T} \mathbf{h}$$

where $\mathbf{J}(\mathbf{x})$ is $\mathbf{f}(\mathbf{x})$'s Jacobian matrix, and $\mu > 0$ is the damped coefficient. Please prove that $L(\mathbf{h})$ is a strictly convex function. (Hint: If a function $L(\mathbf{h})$ is differentiable up to at least second order, L is strictly convex if its Hessian matrix is positive definite.)

2. (Programming) I have established a dataset for training models for detecting speed-bumps and persons. This dataset can be downloaded from <u>https://github.com/csLinZhang/CVBook/tree/main/chapter-15-YOLO/For-yolov4</u>. Using this dataset, please train a speed-bump detection model and test your model on the provided test video (on the course website). For this question, you only need to hand in your video with detected bounding-boxes to the TA. A sample

frame of our result video may like the following image.



3. (Experiment) 3D face scan and editing. Please refer to the files on the course website.